Brane Universes and the Cosmological Constant

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Abstract

The cosmological constant problem and brane universes are reviewed briefly. We discuss how the cosmological constant problem manifests itself in various scenarios for brane universes. We review attempts – and their difficulties – that aim at a solution of the cosmological constant problem.

1 Introduction

The smallness of the cosmological constant (or vacuum energy, or dark energy) is one of the major puzzles of present day cosmology. The fact that recent astronomical observations can be interpreted as evidence for a nonvanishing cosmological constant does by no means resolve the huge discrepancy between its measured value, and its value computed in field theoretical models of particle physics. For recent reviews of the subject see refs. [1–9].

In the present paper we review different approaches to brane universes, and discuss how the cosmological constant problem presents itself. To this end we first have to clarify the nature of the problem in the context of standard 4-dimensional cosmology (since later, in the context of brane worlds, the observed acceleration rate of our universe is no longer proportional to the vacuum energy located on the brane we live on). We also give a short introduction to brane universes; here we will be far from complete, but introduce just the formalisms required subsequently.

Then we review several brane world scenarios, that differ in the way how the 4-dimensional behaviour of gravity is ensured (at least over the tested range of length scales). We will give the arguments in favour of possible solutions of the cosmological constant problem in some of these models, but later we have to conclude that none of these attempts proves to be successful at present.

2 The Cosmological Constant Problem

The origin of the cosmological constant problem is the application of Einstein's equations for the metric $g_{\mu\nu}(x)$ to the cosmological evolution of our universe, that otherwise gives rise to the very successful cosmological standard model. These equations involve the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar R constructed from the metric:

$$R_{\mu\nu} - \frac{1}{2} \, g_{\mu\nu} \, R = -\kappa \, T_{\mu\nu} \qquad (2.1)$$

with $\kappa = 8\pi\mathcal{G}/c^2 \cong 1,865 \cdot 10^{-29}$ m/g. $T_{\mu\nu}$ in (2.1) is the energy-momentum tensor of matter which acts as source for the gravitational field. Well-known vacuum solutions of eq. (2.1) (with $T_{\mu\nu} = 0$) are the Schwartzschild solution, that gives rise to the gravitational attraction between massive objects, and gravitational waves, that await their discovery.

In order to apply eq. (2.1) to the cosmological evolution of our universe one can use the observational fact that, at cosmological scales, the universe can be considered as homogenous and isotropic. Then matter can be modelled by a (possibly relativistic) homogenous and isotropic perfect fluid with matter density $\rho(t)$ ($=T_{00}(t)$) and pressure $p(t)$ ($=T_{ii}(t)$, no sum over i). The equation of state of a perfect fluid determines a relation $p = p(\rho)$. Usually one assumes $p = w\rho$, where the constant w depends on the microscopic properties of the fluid:

for nonrelativistic matter one has $w \simeq 0$, whereas for relativistic matter (radiation) one has $w \simeq \frac{1}{3}$.

For the metric $g_{\mu\nu}(x)$ on can make the Robertson-Walker ansatz

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right) . \tag{2.2}$$

Here the constant k can be chosen, after an appropriate rescaling of r and $a(t)$, as $k = 0, \pm 1$. It determines the global geometry of three-dimensional space, which is flat for $k = 0$, a three-dimensional hypersphere (and hence closed) for $k = 1$, and hyperbolic (open) for $k = -1$. $a(t)$ in (2.2) is the scale factor of the three-dimensional space, and its time dependence is determined by the 00 or ii components of Einstein equations (2.1):

$$3 \, \frac{\dot{a}^2 + k}{a^2} = \kappa \, \rho(t) , \tag{2.3a}$$

$$- \frac{2a\ddot{a} + \dot{a}^2 + k}{a^2} = \kappa \, p(t) \tag{2.3b}$$

where $\dot{a} = da/dt$. (Evidently eqs. (2.3) imply some relation between ρ and p which implies, with $p = w\rho$, $\dot{\rho} + 3 \, \frac{\dot{a}}{a}(1 + w)\rho = 0$ corresponding to energy conservation.)

In general, once the dynamics of matter (in the form of fields) is described by a Lagrangian $\mathcal{L}_M$ minimally coupled to the metric $g_{\mu\nu}$ (in order to ensure invariance under general coordinate transformations), the energy momentum tensor is obtained as the variation of $\mathcal{L}_M$ with respect to $g_{\mu\nu}$:

$$T_{\mu\nu}(x) = \frac{2}{\sqrt{-g(x)}} \frac{\delta}{\delta g^{\mu\nu}(x)} \int d^4x' \, \mathcal{L}_M(x') . \tag{2.4}$$

In the case of homogenous and isotropic field configurations, the general structure of $T_{\mu\nu}(x)$ is then

$$T_{00} = \rho(t) + \Lambda \quad , \qquad T_{ii} = p(t) - \Lambda . \tag{2.5}$$

Typical contributions to the (cosmological) constant Λ arise from the effective (classical or quantum) potential in $\mathcal{L}_M$,

$$\Lambda = V_{eff}(\widehat{\phi}) , \tag{2.6}$$

where $\widehat{\phi}$ are the fields at the minimum of V_{eff}. Now eqs. (2.3) are replaced by

$$3 \, \frac{\dot{a}^2 + k}{a^2} = \kappa \, (\rho(t) + \Lambda) , \tag{2.7a}$$

$$- \frac{2a\ddot{a} + \dot{a}^2 + k}{a^2} = \kappa \, (p(t) - \Lambda) . \tag{2.7b}$$

These equations have to be confronted with astronomical measurements, notably i) the Hubble constant $H_0 = \frac{\dot{a}}{a}|_{today}$ = redshift vs. distance, ii) the present acceleration $\frac{\ddot{a}}{a}$ of the

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universe (from distant supernovae counting), iii) the microwave background anisotropy and iv) dynamical matter measurements. Details of the comparison of these measurements with eqs. (2.7) are discussed in refs. [1–9], here we will just present the results:

First, all measurements are compatible with a flat universe ($k = 0$), and the absence of a large component of radiation ($p(t) \simeq 0$ in eq. (2.7b)). For the present Hubble constant, non-relativistic matter density ρ_0 and the cosmological constant Λ the best fits give

$$H_0 \simeq 65 \frac{km}{sec}(Mpc)^{-1} \ , \quad \rho_0 \simeq 3 \cdot 10^{-24} \frac{g}{m^3} \ , \quad \Lambda \simeq 6 \cdot 10^{-24} \frac{g}{m^3} \sim (2.3 \cdot 10^{-3} eV)^4 \ . \quad (2.8)$$

Whereas the value for ρ_0 indicates that there seems to be more dark matter than visible matter (in agreement with other observations), even the order of magnitude of Λ is difficult to understand: From the standard model of particle physics we expect $\Lambda \sim (1 \ GeV)^4$ from QCD, $\Lambda \sim (100 \ GeV)^4$ from the Higgs potential, and even $\Lambda \sim (10^{18} \ GeV)^4$ from quantum contributions to the vacuum energy, with an ultraviolet cutoff of the order of the Planck scale. This latter value can be improved in the case of supersymmetric extensions of the standard model of particle physics: If supersymmetry is broken at a scale $M_{SUSY} \sim 100 \ GeV$, quantum corrections to the vacuum energy give $\Lambda \sim (100 \ GeV)^2 \cdot (10^{18} \ GeV)^2$ or $\Lambda \sim (100 \ GeV)^4$, depending on the way supersymmetry breaking manifests itself in the particle spectrum (most models would lead to the first larger value; for a discussion and alternatives see, e.g., ref. [10]). In any case, the discrepancy between these values and the one given in eq. (2.8) is enormous, which is what one denotes as the cosmological constant problem.

An important point is to be made here, however: the "observed" value of Λ is deduced from eqs. (2.7), essentially from the measurement of the present acceleration $\frac{\ddot{a}}{a}$ of the universe. Hence, possible solutions to this puzzle could be related to the fact that eqs. (2.7) are too naive. In fact, these equations get modified in higher dimensional universes, notably in brane universes, which are the subject of the next chapter.

3 Brane Universes

The Einstein equations (2.1) can trivially be generalized to space-times with $D > 4$ dimensions. Nowadays there exist two concepts that can nevertheless lead to an effective 4-dimensional behaviour of gravity, at least over the range of length scales (from millimeters to the actual size of the universe) where 4-d gravity agrees with experiment. The traditional concept is the one of Kaluza and Klein, according to which all extra $D - 4$ dimensions are compact. It ensures also a 4-d behaviour of all other fundamental interactions over the range they are tested, provided the size of the extra dimensions is less than $\sim (100 \ GeV)^{-1}$ (in units where $c = \hbar = 1$).

The new concept is the one of brane universes, which are motivated by the presence of D-branes in string theory. Here matter – at least the observed fields of the standard

model of particle physics – live on a $3 + 1$ dimensional hypersurface that is embedded in a larger D-dimensional space-time, traditionally denoted as bulk. (A p-brane is a brane with p spatial dimension, thus our $3 + 1$ dimensional space-time corresponds to a 3-brane.) Now a 4-d behaviour of the other fundamental interactions is guaranteed, but a 4-d behaviour of gravity – at least over the required range of scales – is not automatic. In the subsequent sections of this paper we will review different brane world models that do lead to 4-d gravity, and discuss the cosmological constant problem in each of them.

First, however, we will present briefly how brane worlds are described in terms of a gravitational action and gravitational field equations.

In $D = 4 + N$ space-time dimensions we will split the coordinates into x^μ, $\mu = 1 \ldots 4$, and y^α, $\alpha = 1 \ldots N$. The indices of the metric tensor are split correspondingly, hence $g^{(4+N)}$ has indices $g_{\mu\nu} = g^{(4)}_{\mu\nu}$, $g_{\mu\alpha}$ and $g_{\alpha\beta}$.

For our subsequent purposes it is convenient to assume that the brane is located at $y^\alpha = 0$. (This assumption is not invariant under general coordinate transformations, but corresponds to certain "gauge". This fact has to be considered carefully once one wants to identify the full set of physical fluctuations of the metric. Also, in the case of inhomogenously distributed matter on the brane, this is not necessarily the most convenient choice of gauge [11].)

In the action below we will assume that the bulk is empty, up to a (cosmological) constant Λ_{Bulk}. On the brane we will allow for a general Lagrangian $\mathcal{L}_M$, that can include a (different) cosmological constant Λ as well. Then the action reads

$$S = \int d^4 x d^N y \left[\sqrt{-g^{(4+N)}} \left(\frac{1}{2\kappa_B} R^{(4+N)} + \Lambda_{Bulk} \right) + \sqrt{-g^{(4)}} \delta^N(y^\alpha) \mathcal{L}_M \right] \qquad (3.1)$$

where κ_B is the "fundamental" gravitational coupling constant in the bulk.

The effect of the "brane" term $\sim \delta^N(y^\alpha)$ on the gravitational field equations can most easily be studied by replacing $\mathcal{L}_M$ by Λ, and assuming one extra dimension y only. Then the components $\mu\nu$ of the Einstein equations (2.1) become

$$R^{(4+1)}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{(4+1)} = \kappa_B \left(g_{\mu\nu} \Lambda_{Bulk} + \frac{1}{\sqrt{g_{yy}}} \delta(y) g_{\mu\nu} \Lambda \right). \qquad (3.2)$$

In order to match the Dirac δ-function on the right hand side of eq. (3.2) it is useful to recall that its left hand side, to linear order in $g_{\mu\nu} - \eta_{\mu\nu}$, reads $\Box^{(4)} g_{\mu\nu} + \partial_y \partial^y g_{\mu\nu}$. The second derivatives w.r.t. y generate a corresponding δ-function provided that

$$\partial_y g_{\mu\nu}(y = +\epsilon) = \partial_y g_{\mu\nu}(y = -\epsilon) + \frac{\kappa_B \Lambda}{2\sqrt{g_{yy}}} g_{\mu\nu} . \qquad (3.3)$$

Hence, the first derivative of $g_{\mu\nu}$ w.r.t. y has to jump across the brane, by an amount depending on the cosmological constant Λ on the brane (which corresponds to what is also called the brane tension).

How does the cosmological constant problem represent itself in brane universes? First we have to recall that the values for both H_0 and Λ in (2.8), deduced from eqs. (2.7), are tiny compared to "fundamental" scales $(1\ GeV)^{-1}$ or $(100\ GeV)^{-1}$ in particle physics. The reason therefore are the relatively tiny measured values for the expansion rate $\frac{\dot{a}}{a}$ and the acceleration $\frac{\ddot{a}}{a}$ of the universe. Hence, as compared to fundamental scales in particle physics, our universe can be considered as practically static (time independent). Hence realistic brane universes, apart from leading to an effective 4-d behaviour of gravity, should allow for a time independent scale factor on the brane in zeroth approximation. Only subsequently one has to check, whether a relatively small amount of additional matter and vacuum energy on the brane generates a relatively slow cosmological evolution in agreement with observations.

Now recall that both classical and quantum fields confined to a brane tend to generate a "large" cosmological constant Λ *on the brane*. Is it possible that static brane worlds exist for an *arbitrary* cosmological constant Λ on the brane? This possibility is not excluded, given lower dimensional examples: The Schwartzschild solution in $d = 4$ can be interpreted as a 0-brane in a 4-dimensional bulk, and it is know to be both static and stable for arbitrary mass parameter m that plays the role of Λ on the brane. Also, the cosmic string solution in $d = 4$ can be considered as a 1-brane embedded in a higher dimensional bulk, and is static for arbitrary string tension (but possibly unstable [12]). Naive extrapolations of these configurations to $4 + N$ dimensional brane worlds have, however, to be modified in order to generate an effective 4-d behaviour of gravity. As we will discuss below, these realistic scenarios do no longer seem to allow for an arbitrary value of Λ.

4   Effective 4-d Gravity versus the Effective Cosmological Constant

Effective 4-d gravity corresponds to an (asymptotic) gravitational potential $V(r) \sim \frac{1}{r}$, and $not\ V(r) \sim \frac{1}{r^{1+N}}$ in a $D = 4+N$ dimensional brane world. The desired gravitational potential is generated by the exchange of a massless graviton, that satisfies the *four dimensional* wave equation. Let us now consider various brane world scenarios that lead to such a gravitational field equation.

4.1   One compact extra dimension ($D = 5$)

Here the only extra dimension y is considered as periodic, i.e. the points y and $y + 2\pi R$ are identified and the metric (and its derivatives) have to satisfy corresponding periodicity conditions in y. The search for corresponding cosmological solutions of Einstein's equations is greatly simplified by the fact that due to the assumptions of homogeneity *and* time independence the metric $g_{\mu\nu}$ depends on y only. Hence Einstein's equations for $g_{\mu\nu}$ become

a simple second order differential equation in y where, however, both the boundary condition (3.3) and the periodicity condition have to be imposed. One finds rapidly that these conditions are in conflict [13] unless a) a second brane with tension $\Lambda_2 = -\Lambda$ is introduced, and b) the brane cosmological constant Λ, the bulk cosmological constant Λ_{Bulk} and the 5-d gravitational constant κ_B are related via

$$\Lambda_{Bulk} = -\frac{1}{6}\kappa_B^2 \Lambda^2 \ . \tag{4.1}$$

For nonvanishing Λ_{Bulk} and Λ satisfying (4.1) this brane world is named Randall-Sundrum scenario I [14].

The way 4-d gravity emerges in this scenario is fairly straightforward due to the compactness of the extra dimension: First, one has to decompose the metric into a background metric $g^0_{\mu\nu}(y)$ and fluctuations $h_{\mu\nu}$:

$$g_{\mu\nu}(x,y) = g^0_{\mu\nu}(y) + h_{\mu\nu}(x,y) \tag{4.2}$$

Then, the linearized Einstein's equations (in $h_{\mu\nu}$) become a wave equation for $h_{\mu\nu}$:

$$\Box^{(4)}h_{\mu\nu} + O^{(y)\rho}{}_{\mu\nu}{}^{\sigma}h_{\rho\sigma} = 0 \tag{4.3}$$

with

$$O^{(y)\rho}{}_{\mu\nu}{}^{\sigma}h_{\rho\sigma} = \partial_y\partial^y h_{\mu\nu} - 2R^{(0)\rho}{}_{\mu\nu}{}^{\sigma}h_{\rho\sigma} \ , \tag{4.4}$$

where $R^{(0)\rho}{}_{\mu\nu}{}^{\sigma}$ is the Riemann tensor constructed from the background metric $g^0_{\mu\nu}(y)$.

For a compact extra dimension the spectrum of $O^{(y)}$ is semi-positive and discrete: $O^{(y)}h^{(n)}_{\mu\nu} = k_n^2 h^{(n)}_{\mu\nu}$ with $k_0^2 = 0$, $k_n^2 > 0$ for $n \geq 1$. The zero mode $h^0_{\mu\nu}$ represents the massless 4-d graviton (obeying the wave equation $\Box^{(4)}h^0_{\mu\nu} = 0$), and its exchange generates the desired 4-d gravitational potential $V(r) \sim \frac{1}{r}$.

On the other hand, the compactness of the extra dimension was also responsable for the two constraints $\Lambda_2 = -\Lambda$ and eq. (4.1) above. Recall that our present assumption of a time independent background metric $g^0_{\mu\nu}$ corresponds to a vanishing effective 4-d cosmological constant (eqs. (2.7) with $k = 0$ and vanishing right hand sides). Now this assumption does not require a vanishing cosmological constant on the brane(s), but instead two fine tuning conditions on the fundamental parameters of the model. In fact, if both of these conditions are violated, one obtains not only a rapidly accelerating scale factor on the brane(s), but also a time dependent effective 4-d gravitational constant in strong disagreement with observations. Hence the cosmological constant problem is far from being solved, in some sense the need to satisfy two fine tuning conditions among the fundamental parameters is even worse than before. As discussed in [15] this remains valid after adding scalar fields with arbitrary Lagrangian in the bulk.

4.2 One Non-Compact Extra Dimension ($D = 5$)

In [16] it has been proposed to let the size of the extra dimension go to infinity, whereupon it becomes non-compact. The periodicity condition (in y) on the metric is now replaced by the condition that $g_{\mu\nu}(y)$ remains finite for $y \to \pm\infty$. Amazingly, 4-d gravity still emerges down to sufficiently small length scales:

Now, the spectrum of the operator $O^{(y)}$ in (4.3) and (4.4) is continuous, but the corresponding eigenfunctions are not normalizable with the exception of one normalizable zero mode. As shown in [16], the existence of this normalizable zero mode is sufficient to generate 4-d gravity down to sufficiently small length scales (with deviations at small distances that are possibly measurable in the future).

Does the absence of periodicity conditions on the metric lead to the absence of the two fine tuning conditions before? Unfortunately not, since the condition that $g_{\mu\nu}(y)$ remains finite for $y \to \pm\infty$ still requires a particular relation among the fundamental parameters, that turns out to be identical to eq. (4.1). Hence the number of fine tuning conditions is reduced from 2 to 1 (due to the absence of the second brane), but the remaining condition (4.1) is still required in order to tune the effective 4-d cosmological constant to zero.

4.3 Several Non-Compact Extra Dimensions (the DGP Model)

A scenario for effective 4-d gravity with non-compact extra dimensions, but no cosmological constant in the bulk, has been proposed in [17, 18, 19]. Instead, an additional Einstein term (that includes a kinetic term for the graviton) is added to the action on the brane. Replacing $\mathcal{L}_M$ in (3.1) by Λ and omitting the cosmological constant in the bulk, the action is given by

$$S = \int d^4x d^N y \left[\sqrt{-g^{(4+N)}} \frac{1}{2\kappa_B} R^{(4+N)} + \sqrt{-g^{(4)}} \delta^N(y) \left(\frac{1}{2\kappa_4} R^{(4)} + \Lambda \right) \right] \qquad (4.5)$$

which involves two different gravitational couplings κ_B (in the bulk) and κ_4 (on the 3+1 dim. brane). It can even be argued that it would be unnatural to omit the Einstein term $R^{(4)}$ on the brane, since matter induced quantum corrections would generate it anyhow. In [17, 18] the above model has been formulated in $D = 5$ dimensions ($N = 1$), but an interesting result with respect to the cosmological constant problem emerges only in $D \geq 6$ dimensions ($N \geq 2$) as considered in [19].

The action (4.5) leads to an interesting structure for the graviton propagator $G(p, |y-y'|)$, where p is the 4-d momentum (parallel to the brane), and y, y' are arbitrary end points in the bulk. (Here we neglect for simplicity the tensorial structure of the graviton propagator.) Writing $\kappa_4^{-1} = M_{Pl}^2$, $\kappa_B^{-1} = M_B^{N+2}$ (in $D = 4 + N$ dimensions), one finds for $N > 2$

$$G(p, |y-y'|) \simeq \frac{|y-y'|^{2-N} M_B^{N-2}}{M_B^2 + p^2 M_{Pl}^2 |y-y'|^{2-N} M_B^N} \ . \qquad (4.6)$$

We recall that 4-d gravity requires a "brane-to-brane" propagator $G(p, 0)$ that behaves like $\frac{1}{p^2}$. The limit $|y - y'| \to 0$ in eq. (4.6) is obviously singular for $N > 2$. (A detailed study for $N = 2$ reveals the appearance of logarithmic singularities in this case.) A naive regularization consists in replacing $|y - y'|^{2-N}$, for $|y - y'| \to 0$, by M_B^{N-2}, the (fundamental) gravitational constant in the bulk. Then one obtains

$$G(p, 0) \simeq \frac{1}{M_B^2 + p^2 M_{Pl}^2 / M_B^2} \qquad (4.7)$$

which behaves like $\frac{1}{p^2}$ only for $p^2 \gg M_B^4 / M_{Pl}^2$, i.e. for length scales $r < r_c \sim \frac{M_{Pl}}{M_B^2}$. In order not to mess up the successful predictions of the cosmological standard model, we need $r_c \gtrsim H_0^{-1} \sim 10^{33} \ eV^{-1}$ which corresponds to $M_B \lesssim 10^{-3} \ eV$, indeed a quite unusual value for the "fundamental" (D-dimensional) gravitational scale.

The essential feature of the model is then the modified (massive) behaviour of the graviton propagator at length scales larger than r_c. As a consequence, gravity does not necessarily react to sources that are smooth at scales larger than $r_c \sim H_0^{-1}$, as it is the case for a cosmological constant Λ on the brane. As proposed in [19] and [20] (see ref. [21] for a review), this could lead to a potential solution of the cosmological constant problem.

However, the scenario faces two severe problems: First, less naive UV regularizations of the graviton propagator (by smearing out the previously vanishing width of the brane) tend to be in conflict with the requirement to solve Einstein's equations also at small distances. This can be shown explicitly in a solvable scenario where the previously infinitely thin brane is considered as "hollow", and Einstein's equations are required to be satisfied also inside as well as across the surface [22]. As a consequence, additional constraints on the parameters of the model appear, that re-introduce a fine tuning condition for the existence of a static solution.

Second, a study of all tensorial components of the graviton propagator reveals the presence of negative norm states (with negative residues) [23] which signal an instability of the configuration. Although the details depend on the UV regularization mentionned above, the fact that the effective mass term for the graviton is not of the Fierz-Pauli form renders this problem certainly difficult to solve. Hence it cannot be claimed at present that these scenarios solve the cosmological constant problem.

4.4   Two Compact Extra Dimensions

Scenarios with two "football shaped" compact extra dimension and a time independent metric for an arbitrary value of the cosmological constant on the brane have been proposed in [24, 25]. Due to the compactness of the extra dimensions the emergence of 4-d gravity is evident, but the issue is now whether the junction conditions of the metric on the brane (conditions on its derivatives perpendicular to the brane, in analogy to the condition (3.3) in the case of one co-dimension $N = 1$) can be satisfied for Λ arbitrary.

This requires a particular form of the curvature in the bulk, which is induced by additional matter in the form of a $U(1)$ gauge field with field strength F_{MN} in the bulk. A configuration of F_{MN} (with indices in the extra dimensions) that solves the gauge field equations of motion, generates the required curvature and is adopted to the singular geometry at the position of the brane corresponds to the one of a magnetic monopole, leading to an arbitrary value of the deficit angle α (w.r.t. 2π) of a circle that encloses the brane in the two extra dimensions. A priori α can be chosen such that the junction conditions are satisfied for any value of Λ, and then the scale factor on the brane is time independent for any value of Λ, which seems to solve the cosmological constant problem.

However, as noted in [25, 26], the magnetic flux corresponding to F_{MN} has to satisfy a Dirac quantization condition such that it is an integer devided by two times the gauge coupling. It follows that only discrete values of the deficit angle α, and hence of Λ are allowed. This – and the instability with respect to perturbations [27] – rules out the possibility to solve the cosmological constant problem for arbitrary (notably time dependent, as near the end of an inflationary epoch) values of Λ.

4.5   Self Tuning Models

In fact, this previous scenario is just a particular (though a priori quite promising) case of so-called self tuning models, that are reviewed in [28]. Their common feature is the presence of additional fields in the bulk, whose "vacuum" configurations can be arranged such that all equations of motion are satisfied, and the metric and all fields (and hence the scale factor on the brane) are time independent, for arbitrary values of Λ.

The first models of this kind (with $N = 1$ extra dimension) involved a dilaton like scalar field, present also in the bulk, with an exponential potential on the brane [29]. However, in the case of non compact extra dimensions these models lead generically to metrics that are singular at infinity. As shown in [30], regularization of these singularities re-introduces fine tuning.

In the case of compact extra dimensions, flux quantization conditions (as the one above) allow generically just for discrete sets of parameters. This does not allow for reasonable 4-d physics, as clarified in [28]. In addition, particular (fine tuned) cosmological initial conditions are required in these scenarios, which corresponds just to a shift of the cosmological constant problem and not to its solution.

5   Outlook

It is certainly true that brane worlds offer new tools that could potentially solve the cosmological constant problem. However, from a 4-d point of view, it seems that a solution of the problem is practically impossible [1] as long as neither gravity itself, nor the way

gravity reacts to energy-momentum, is modified. Note that modifications at small distances (as expected anyhow, if gravitational quantum UV divergencies are regularized) are useless here, since the problem – the very slow evolution of the universe as compared to fundamental scales in particle physics – concerns physics at very large distances (or time scales).

On the other hand, any brane world scenario that is proposed as a solution of the problem must also lead to an effective 4-d theory of gravity and its coupling to matter (which is, after all, what we see). It seems that such an effective 4-d theory must have unconventional properties at (very) large distances. It is notoriously difficult to tamper with the infrared behaviour of gauge theories such as general relativity without generating inconsistencies as negative norm states, tachyons, or violations of unitarity and/or causality. Nevertheless it is not excluded that, by looking for solutions of the cosmological constant problem, further studies of brane worlds uncover consistent modifications of 4-d gravity at large distances that lead to a solution of the perhaps most puzzling problem of fundamental physics today.

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